Mathematics Standards Rationale

The four National Council of Teachers of Mathematics (NCTM) standards of problem solving, reasoning, communication and connections are goals interwoven throughout the Arizona mathematics standards. These goals are the reason people study and use mathematics, and they should permeate everything we do in and outside the classroom.

Whenever possible, mathematical learning should be placed in a broader, problem solving context and evaluated through performance assessments. In this setting, students discover questions involving numbers or equations from a real-world context which lead to answers that have meaning. Ultimately, all problems should be application problems; more ideally, problems should be presented in the broader context of an investigation or project. This way the students use problem solving, reasoning, communication and connections in every mathematical activity. The spirit of these four goals is a mathematical apprenticeship in which the students solve problems on a daily basis, much as mathematics is used in the real world.

Even the youngest students can use mathematics to solve social science problems, engineering problems and business problems in a meaningful way. As early as possible, students should learn that mathematics is everywhere in the world around them. They should realize that in the real world not all answers are small whole numbers; instead they can be large or small and/or have a fractional part.

As students develop their ability to perceive and conceptualize in problem solving, they should reason about the mathematics they do. Teachers should guide students to ask such questions as: *Does the answer make sense? Are there other ways to arrive at the answer? Does the answer bring up more questions? Can I answer those? What other information would I need?* It is this kind of reasoning that enriches a mathematical educational experience. If students do answer such significant questions, they then naturally apply mathematics in everyday life. Without this guidance, they remain mathematically deficient.

Teachers should engage students in mathematical discourse at all stages of learning. Mathematics was developed as a means to communicate about quantities, logical relationships and unknowns. To use this language, students should communicate (both orally and in writing) everything they do mathematically. They should explain their mathematical thinking through language, through models, graphically, geometrically, numerically and algebraically. Students should be encouraged to express themselves in as many ways as possible and to learn to translate between one mathematical language and another.

Students should regularly see the mathematical connections within the course of an investigation or project. They must experience mathematics primarily through its connections to other disciplines. For too long we have structured our curricula to reach the few who will use mathematics in isolation rather than the majority who will apply it to their work or study in other fields.

A variety of tools should be available to students as they develop concepts and understandings of mathematics. Graphing calculators and computers should be standard equipment in mathematics classrooms. New technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

As the four essential standards–problem solving, reasoning, communication and connections– and the implementation of technology become functioning parts of our curricula, we can expect all Arizona students will develop the mathematical power to confidently handle the future. They will be able to face the world knowing that they can not only merely compute but also that they can use meaningful mathematics to solve real problems.

The organization of the content in these standards is designed for readability purposes and is not intended to dictate sequence or to define the structure of courses. Topics from all six mathematics standards need to be continuously integrated within the curricula.

Table 1. Mathematics Standards

STANDARD 1: **Number Sense**

Students develop number sense and use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to determine the reasonableness of results.

STANDARD 2: **Data Analysis and Probability**

Students use data collection and analysis, statistics, and probability to make valid inferences, decisions and arguments and to solve a variety of real-world problems.

STANDARD 3: **Patterns, Algebra and Functions**

Students use algebraic methods to explore, model and describe patterns, relationships and functions involving numbers, shapes, data and graphs within a variety of real-world problem solving situations.

STANDARD 4: **Geometry**

Students use geometric methods, properties and relationships as a means to recognize, draw, describe, connect and analyze shapes and representations in the physical world.

STANDARD 5: **Measurement and Discrete Mathematics**

Students make and use direct and indirect measurement, metric and U.S. customary, to describe and compare the real world and to prepare for the study of discrete functions, fractals and chaos which have evolved out of the age of technology.

STANDARD 6: **Mathematical Structure/Logic**

Students use both inductive and deductive reasoning as they make conjectures and test the validity of arguments.

MATHEMATICS STANDARDS
BY LEVEL: PROFICIENCY (Grades 9-12) and DISTINCTION (Honors)

STANDARD 1: NUMBER SENSE

Students develop number sense and use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to determine the reasonableness of results.

(Proficiency Grades 9-12)

- **1M-P1. Compare and contrast the real number system and its various subsystems with regard to their structural characteristics**

Core – will be tested on AIMS

PO 1. Classify numbers as members of the sets (natural, whole, integers, rationals and irrationals)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 2. Compare subsets of the real number system with regard to their properties (commutative, associative, distributive, identity, inverse and closure properties)
PO 4. Identify whether a given set of numbers is finite or infinite
{PO 3 Deleted}

- **1M-P2. Construct, interpret and demonstrate meaning for real numbers and absolute value in problem-solving situations**

Core – will be tested on AIMS

PO 1. Determine a rational estimate of an irrational number
PO 3. Solve real-world distance problems using absolute value
PO 4. Determine, among the solutions to a real-world problem, which, if any, is reasonable
PO 6. Choose the appropriate signed real number to represent a real-world value
PO 7. Use the appropriate form of a real number to express a real-world situation (e.g., choosing between a radical expression or rational approximation)
PO 8. Convert standard notation to scientific notation, including negative exponents, and vice versa

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 2. Define *absolute value* as the distance from the origin

{PO 5 Moved to 4M-P4, PO4}

(Distinction - Honors)

Students know and are able to do all of the above and the following:

- **1M-D1. Develop conceptual understanding of the complex number system**

- **1M-D2. Demonstrate facility with operations in the complex number system**

STANDARD 2: DATA ANALYSIS AND PROBABILITY

Students use data collection and analysis, statistics, and probability to make valid inferences, decisions and arguments and to solve a variety of real-world problems.

(Proficiency Grades 9-12)

- **2M-P1. Construct and draw inferences including measures of central tendency, from charts, tables, graphs and data plots that summarize data from real-world situations**

Core – will be tested on AIMS

PO 1. Organize collections of data into frequency charts, stem-and-leaf plots, scatter plots

PO 2. Construct histograms, line graphs, circle graphs and box-and-whisker plots

PO 4. Evaluate the reasonableness of conclusions drawn from data analysis

PO 5. Use mean, median, mode, quartiles and range as a means for effective decision making in analyzing the data and the outliers

PO 6. Identify graphic misrepresentations and distortions of sets of data (e.g., omissions of parts of axis range, unequal interval sizes)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 3. Draw inferences from collections of data

- **2M-P2. Use appropriate technology (e.g., graphing calculators, computer software) to display and analyze data**

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Use appropriate technology to display data as lists, tables, matrices and plots
PO 2. Use appropriate technology to calculate mean, median, mode, minimum and maximum
PO 3. Use appropriate technology to predict patterns in sets of data (e.g., "Does a scatter plot appear to be linear?")

- **2M-P3. Apply curve fitting to make predictions from data**

Core – will be tested on AIMS

PO 1. Draw a line which closely fits a scatter plot
PO 2. Make a prediction from a linear pattern in plots of data

*Beyond Core**

PO 1. Draw a curve which closely fits a scatter plot

* Beyond Core: Appropriate to be taught after a grounding in core instruction, but will not be tested on AIMS

- **2M-P4. Explain the effects of sampling on statistical claims and recognize misuses of statistics**

Core – will be tested on AIMS

PO 1. Differentiate between sampling and census
PO 2. Differentiate between a biased and an unbiased sample
PO 3. Recognize the impact of interpreting data from a biased sample

Beyond Core

PO 4. Distinguish the effects of using statistical measures obtained from a sample vs. those obtained from a census
PO 5. Recognize the misinterpretations of data from different representations of those same data
PO 6. Determine the validity of sampling methods in studies

- **2M-P5. Design and conduct a statistical experiment to study a problem and interpret and communicate the outcomes**

Beyond Core

PO 1. Design a statistical experiment based on a given hypothesis
PO 2. Create an appropriate data-gathering instrument (e.g., biased vs. unbiased questions, multiple choice vs. open-ended)
PO 3. Organize collected data into an appropriate graphical representation
PO 4. Draw and support inferences that are based on data analysis

- **2M-P6. Use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty**

Beyond Core

PO 1. Recognize whether experimental or theoretical methods were used to calculate a particular probability
PO 2. Use experimental observations to estimate probabilities of entire populations
PO 3. Distinguish between independent and dependent events
PO 4. Solve probability problems involving *and* and *or* statements, with and without replacement

- **2M-P7. Use simulations to estimate probabilities**

Beyond Core

PO 1. Design appropriate simulations to estimate probabilities of real-world situations (e.g., disk toss, cube toss, technological simulations)
PO 2. Use simulations to estimate probabilities of real-world situations

- **2M-P8. Solve real-world problems by using combinations and permutations**

Core – will be tested on AIMS

PO 1. Use a tree diagram or a chart of possible outcomes to count probable outcomes of an event

Beyond Core

PO 2. Determine when to use combinations in counting objects
PO 3. Determine when to use permutations in counting objects
PO 4. Use combinations and permutations to solve real-world problems not requiring the use of formulas

- **2M-P9. Describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed**

Beyond Core

PO 1. Determine if data gathered from a real-world situation fits a normal curve
PO 2. Describe the central tendency characteristics of the normal curve
PO 3. Make simple predictions from data represented on a given normal curve

- **2M-P10. Explain the concept of a random variable**

Beyond Core

PO 1. Distinguish situations where a random variable is needed or used
{PO 2. Deleted}

- **2M-P11. Apply measures of central tendency, variability and correlation**

Core – will be tested on AIMS

PO 1. Apply the concepts of mean, median, mode and range to draw conclusions about data
PO 3. Determine, from a given plot of data, whether it has positive or negative correlation

Beyond Core

PO 2. Draw conclusions about the "spread" of data given the variance and standard deviation (e.g., compare sets of data with the same central tendency, but with different variance)

(Distinction – Honors)

Students know and are able to do all of the above and the following:

- **2M-D1. Transform data to aid in data interpretation and prediction**

- **2M-D2. Test hypotheses using appropriate statistics**

- **2M-D3. Explain the concept of a random variable to generate and interpret probability distributions including binomial, uniform and normal**

- **2M-D4. Apply the concept of a random variable to generate and interpret probability distributions including binomial, uniform and normal**

- **2M-D5. Apply curve fitting to determine the strength of the relationship between two data sets and to make predictions from data**

STANDARD 3: PATTERNS, ALGEBRA AND FUNCTIONS

Students use algebraic methods to explore, model and describe patterns, relationships and functions involving numbers, shapes, data and graphs within a variety of real-world problem solving situations.

(Proficiency Grades 9-12)

- **3M-P1. Model real-world phenomena (e.g., compound interest or the flight of a ball) using functions and relations (e.g., linear, quadratic, sine and cosine, and exponential)**

Core – will be tested on AIMS

PO 2. Describe a real-world situation that is depicted by a given graph

Beyond Core

PO 1. Identify the independent and dependent variables from a real-world situation

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 3. Sketch a graph that models a given real-world situation

- **3M-P2. Represent and analyze relationships using written and verbal explanations, tables, equations, graphs and matrices and describe the connections among those representations**

Core – will be tested on AIMS

PO 3. Determine whether a relation is a function, given the graphical representation

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Express the relationship between two variables using a table, equation, graph and matrix

{PO 2. Deleted}

- **3M-P3. Analyze the effects of parameter changes on functions (e.g., linear, quadratic and trigonometric) using calculators and/or computers**

Beyond Core

PO 1. Use technology to determine changes in the shape and behavior of polynomial functions (of degree 2 or less) when constants and coefficients are varied

- **3M-P4. Interpret algebraic equations and inequalities geometrically and describe geometric relationships algebraically**

Core – will be tested on AIMS

PO 1. Graph a linear equation in two variables
PO 2. Graph a linear inequality in two variables
PO 3. Determine slope and intercepts of a linear equation
PO 4. Write an equation of the line that passes through two given points
PO 5. Determine from two linear equations whether the lines are parallel, are perpendicular or coincide

- **3M-P5. Apply trigonometry to real-life problem situations (e.g., investigate how to find the distance across a river using similar triangles and trigonometric ratios; compare the sine and cosine curves to the curves of sound waves)**

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Use the definitions of trigonometric functions to find the sine, cosine and tangent of the acute angles of a right triangle

Beyond Core

PO 2. Solve simple right-triangle trigonometric equations involving sine, cosine and tangent
PO 3. Use an appropriate right-triangle trigonometric model to solve a real-life problem

- **3M-P6. Perform mathematical operations on expressions and matrices, and solve equations and inequalities**

Core – will be tested on AIMS

PO 1. Simplify numerical expressions using the order of operations, including exponents
PO 2. Evaluate algebraic expressions using substitution
PO 3. Simplify algebraic expressions using distributive property
PO 4. Simplify square roots and cube roots with monomial radicands that are perfect squares or perfect cubes
PO 6. Evaluate numerical and algebraic absolute value expressions
PO 7. Multiply and divide monomial expressions with integer exponents
PO 9. Solve linear equations and inequalities in one variable
PO 10. Solve formulas for specified variables
PO 11. Solve quadratic equations (integral roots only)
PO 13. Solve proportions which generate linear equations
PO 15. Solve systems of linear equations in two variables (integral coefficients and solutions)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 5. Calculate powers and roots of real numbers, both rational and irrational, using technology
PO 14. Solve absolute value equations containing a single absolute value expression

Beyond Core

PO 8. Add, subtract and perform scalar multiplication with matrices
PO 12. Solve radical equations involving one radical (restrict to square roots)

- **3M-P7. Translate among tabular, symbolic and graphical representations of functions**

Core – will be tested on AIMS

PO 1. Create a linear equation from a table of values
PO 2. Create a graph from a table of values
PO 3. Determine the solution to a system of equations in two variables, from a given graph

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 4. Determine the solution to a system of inequalities in two variables, from a given graph (e.g., “Which of the shaded regions represents the solution to the system?”)

- **3M-P8. Use the power of mathematical abstraction and algebraic symbolism to represent various situations**

  ### *Core – will be tested on AIMS*

  PO 1. Translate verbal expressions and sentences to mathematical expressions and sentences

  PO 2. Generate an algebraic sentence to model real-life situations, given a data set (limited to linear relationships)

- **3M-P9. Determine maximum and minimum points of a graph and interpret results in problem situations**

  ### *Core – will be tested on AIMS*

  PO 2. Determine domain and range of a relation, given the graph or a set of points

  {PO 1 and PO 3 Deleted}

- **3M-P10. Investigate the limiting process by examining infinite sequences and series and areas under curves**

  ### *Beyond Core*

  PO 1. Compare the estimates of the area under a curve over a bounded interval, using progressively smaller rectangles (not using calculus)

  PO 2. Estimate the limit of a given infinite sequence (e.g., given the sequence $1/n$, as n gets larger) (not using calculus)

(Distinction – Honors)

Students know and are able to do all of the above and the following:

- **3M-D1. Use matrices to solve linear systems**

- **3M-D2. Demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations**

- **3M-D3. Understand operations on, and the general principles and behavior of, classes of functions (including logarithmic functions)**

- **3M-D4. Apply general graphing techniques to trigonometric functions**

- **3M-D5. Solve trigonometric equations and verify trigonometric identities**

- **3M-D6. Understand the connections between trigonometric functions and polar coordinates, complex numbers and series**

- **3M-D7. Understand the conceptual foundations of limits, the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines**

- **3M-D8. Analyze the graphs of polynomial, rational, radical and transcendental functions**

STANDARD 4: GEOMETRY

Students use geometric methods, properties and relationships as a means to recognize, draw, describe, connect and analyze shapes and representations in the physical world.

(Proficiency Grades 9-12)

- **4M-P1. Interpret and draw three-dimensional objects**

Core – will be tested on AIMS

PO 1. Sketch prisms, pyramids, cones, cylinders and spheres
PO 2. Classify prisms, pyramids, cones, cylinders and spheres by base shape and lateral surface shape
PO 3. Recognize the three-dimensional figure represented by a two-dimensional drawing (e.g., "What figures are represented by given nets, sketches, photographs?")

- **4M-P2. Represent problem situations with geometric models and apply properties of figures**

Core – will be tested on AIMS

PO 1. Calculate surface areas and volumes of three-dimensional geometric figures, given the required formulas
PO 2. Solve applied problems using angle and side length relationships
PO 3. Solve applied problems using the Pythagorean theorem (e.g., determine whether a wall is square)
PO 4. Solve applied problems using congruence and similarity relationships of triangles (e.g., estimate the height of a building, using shadows)
PO 6. Determine the distance and midpoint between points within a coordinate system representative of a practical application
PO 7. Find the area of a geometric figure composed of a combination of two or more geometric figures, given an appropriate real-world situation and the formulas
PO 8. Solve problems involving complementary, supplementary and congruent angles

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 5. Make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object (models and representations include scale drawings, perspective drawings, blueprints or computer simulations)

- **4M-P3. Deduce properties of figures using transformations in coordinate systems, identifying congruency and similarity**

Core – will be tested on AIMS

PO 1. Determine whether a planar figure is symmetric with respect to a line
PO 3. Determine the effects of a transformation on linear and area measurements of the original planar figure
PO 4. Sketch the planar figure that is the result of a given transformation

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 2. Give the new coordinates of a transformed geometric planar figure

- **4M-P4. Deduce properties of, and relationships between, figures from given assumptions**

Core – will be tested on AIMS

PO 1. Find similarities and differences among geometric shapes and designs using a given attribute (e.g., height, area, perimeter, diagonals and angle measurements)
PO 2. Identify arcs, chords, tangents and secants of a circle
PO 3. State valid conclusions using given geometric definitions, postulates and theorems
PO 4. Represent *pi* as the ratio of circumference to diameter {Moved from 1M-P2, PO 5}

- **4M-P5. Translate between synthetic and coordinate representations (e.g., a straight line is represented by the algebraic equation $Ax + By = C$)**

Core – will be tested on AIMS

PO 1. Determine the relative placement of two lines on a coordinate plane by examining the algebraic equations representing them

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 2. Verify characteristics of a given geometric figure using coordinate formulas such as distance, mid-point, and slope to confirm parallelism, perpendicularity and congruency

- **4M-P6. Recognize and analyze Euclidean transformations (e.g., reflections, rotations, dilations and translations)**

Core – will be tested on AIMS

PO 1. Classify transformations based on whether they produce congruent or similar non-congruent figures
PO 2. Determine whether a given pair of figures on a coordinate plane represents a translation, reflection, rotation and/or dilation

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 3. Apply transformational principles to practical situations (e.g., enlarge a photograph)

(Distinction – Honors)

Students know and are able to do all of the above and the following:

- **4M-D1. Deduce properties of figures using vectors**

- **4M-D2. Apply transformations, coordinates and vectors in problem solving**

STANDARD 5: MEASUREMENT AND DISCRETE MATHEMATICS

Students make and use direct and indirect measurement, metric and U.S. customary, to describe and compare the real world and to prepare for the study of discrete functions, fractals and chaos which have evolved out of the age of technology.

(Proficiency Grades 9-12)

- **5M-P1. Represent problem situations using discrete structures such as finite graphs, matrices, sequences and recurrence relations**

  ### *Beyond Core*
  PO 1. Use matrices and finite graphs to display data
  PO 2. Find a specified n^{th} term of a simple arithmetic or geometric sequence, where the common difference or common ratio is an integer and $n > 100$
  PO 3. Use simple or basic recursion formulas to solve real-life problems (e.g., compound interest)

- **5M-P2. Represent and analyze finite graphs using matrices**

  ### *Beyond Core*
  PO 1. Interpret data using matrices and finite graphs (e.g., networks, street diagrams, tournament schedules, production schedules)
  PO 2. Determine when a finite graph gives an accurate picture of a data set
  PO 3. Translate a finite graph into a matrix and vice versa

- **5M-P3. Develop and analyze algorithms**

  ### *Core – will be tested on AIMS*
  PO 2. Determine the purpose of a given algorithm (simple, basic **math** algorithm)
  PO 3. Determine whether given algorithms are equivalent (simple, basic **math** algorithms)

  ### *Core – to be taught in grades 9-10, but will not be tested on AIMS*
  PO 1. Write an algorithm that explains a particular mathematical process (e.g., tell a younger child how to find the average of two numbers)

- **5M-P4. Solve enumeration and finite probability problems**

Core – will be tested on AIMS

PO 1. Find the outcome set of a situation
PO 2. Find the probability that a specific event will happen
PO 4. Determine the number of possible outcomes in a real-world situation using the counting principle and tree diagrams

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 3. Determine theoretical geometrical probabilities, given necessary formulas (e.g., "Given a circular target on a square base, what is the probability of hitting the circle with a dart, providing the dart goes inside the square?")
{PO 5 Deleted}

(Distinction – Honors)

Students know and are able to do all of the above and the following:

- **5M-D1. Represent and solve problems using linear programming and difference equations**

- **5M-D2. Investigate problem situations that arise in connection with computer validation and the application of algorithms**

- **5M-D3. Describe, analyze and extend patterns produced by processes of geometric change such as fractals**

STANDARD 6: MATHEMATICAL STRUCTURE/LOGIC

Students use both inductive and deductive reasoning as they make conjectures and test the validity of arguments.

(Proficiency Grades 9-12)

- **6M-P1. Use inductive and deductive logic to construct simple valid arguments**

Core – will be tested on AIMS

PO 2. Produce a valid conjecture using inductive reasoning by generalizing from a pattern of observations (e.g., if $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, make a conjecture)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Construct a simple informal deductive proof (e.g., write a proof of the statement: "Given an airline schedule with cities and flight times, you can fly from Bombay to Mexico City")

- **6M-P2. Determine the validity of arguments**

Core – will be tested on AIMS

PO 2. Draw a simple valid conclusion from a given *if* . . . *then* statement and a minor premise
PO 3. Distinguish valid arguments from invalid arguments
PO 4. List related *if* . . . *then* statements in logical order

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Determine if the converse of a given statement is true or false
PO 6. Analyze assertions about everyday life by using principles of logic (e.g., examine the fallacies of advertising)

Beyond Core

PO 7. Recognize the difference between a statement verified by mathematical proof (i.e., a theorem) and one verified by empirical data (e.g., women score higher than men on vocabulary tests)
{PO 5 Deleted}

- **6M-P3. Formulate counterexamples and use indirect proof**

Core – will be tested on AIMS

PO 1. Construct a counterexample to show that a given invalid conjecture is false (e.g., Nina makes a conjecture that $x^3 > x^2$ for all values of x. Find a counterexample.)

- **6M-P4. Make and test conjectures**

Beyond Core

PO 1. Write an appropriate conjecture given a certain set of circumstances
PO 2. Test a conjecture by constructing a logical argument or a counterexample

- **6M-P5. Understand the logic of algebraic procedures**

Core – will be tested on AIMS

PO 1. Determine whether a given algebraic expression and a possible simplified form are equivalent (e.g., show that $(x + y)^2 = x^2 + y^2$ is invalid)
PO 2. Determine whether a given procedure for solving an equation is valid

(Distinction – Honors)

Students know and are able to do all of the above and the following:

- **6M-D1. Prove elementary theorems within various mathematical structures**
- **6M-D2. Develop an understanding of the nature and purpose of axiomatic systems**
- **6M-D3. Construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction**

MATHEMATICS GLOSSARY

Absolute Value A number's distance from zero on a number line. The absolute value of –4 is 4; the absolute value of 4 is 4.

Algebraic Methods The use of symbols to represent quantities and signs to represent their relationships.

Algebraic Sentence A general term for equations and inequalities.

Algorithms A mechanical procedure for performing a given calculation or solving a problem through step-by step procedures such as those used in long division.

Angle Measure The measure of the space between two lines that meet in a point. Angles are measured in degrees or radians.

Axiomatic Systems Systems that include self-evident truths; truths without proof and from which further statements, or theorems, can be derived.

Binomial In algebra, an expression consisting of two terms connected by a plus or minus sign, such as $4a + 6$.

Box-and-Whisker Plot A graphic method for showing a summary of data using median, quartiles and extremes of data. A box plot shows where the data are spread out and where they are concentrated.

Census The count of a population.

Combinations Subsets of a larger number of items (e.g., the number of different teams of three that can be chosen from a group of 21).

Computational Techniques Operations or tools–number lines, calculators.

Complex Numbers Numbers that have the form $a + bi$ where a and b are real numbers and i is an imaginary number.

Congruence The state of having the same size and shape.

Conjecture An inference drawn from observed patterns in several examples.

Contextual Situation Relating mathematical problems to real, modeled or illustrated circumstance.

Coordinate System Any set of two or more magnitudes used to locate points, lines or curves. Commonly placed by using a horizontal axis (x-axis) and vertical axis (y-axis).

Correlation Coefficient A statistical measure that relates how well a set of data points can be modeled by a line.

Cosine The trigonometric function that is defined as the ratio of the leg adjacent to an angle to the hypotenuse of its right triangle.

Counterexample An example of a conditional statement in which the hypothesis is true and the conclusion is false.

Curve Fitting Plotting data and observing the pattern to predict trends.

Deductive Reasoning A series of logical steps in which a conclusion is drawn directly from a set of statements that are known or assumed to be true.

Dilation A transformation that either enlarges or reduces a geometric figure proportionately.

Direct Proof A conclusion proved through deductive reasoning.

Discrete Math The study of mathematical properties of sets and systems that have only a specific number of elements. For example, the results of tossing dice form a discrete set of events, since a die has to land on one of its six faces.

Empirical Relating to the collection of actual data.

Equation A mathematical statement in which one expression is equal to another.

Euclidean Transformations In geometry, the process of changing one configuration into another, including slides, rotations and reflections.

Exponent Tells how many times a number or variable is used as a factor. For example, 6 with an exponent of 3 (6^3) indicates that 6 is a factor 3 times (6 x 6 x 6).

Exponential Function A function commonly used to study growth and decay. It has a form $y = a^x$.

Expression A mathematical phrase with no equal sign, such as $3x$, 6, $2n + 3m$.

Factors Any of two or more quantities that are multiplied together.

Finite Graph A structure consisting of vertices and edges, where the edges indicate a mapping among the vertices (e.g., the vertices may represent players in a tournament, and the edges indicate who plays whom).

Flip A transformation, also called a reflection, that produces a mirror image of a geometric figure.

Fractal An algebraically generated complex geometric shape having the property of being endlessly self-similar under magnification. Some computer screen savers utilize fractals.

Function A dependent relationship between two sets of numbers in which a value in the first set has only one defined element in the second set.

Identify To state, match, select, write.

Imaginary Numbers The square root of a negative number usually expressed using *i*, e.g., $(\sqrt{-9}) = 3i$.

Indirect Proof A deductive proof using contradiction or elimination to rule out all except the desired conclusion.

Inductive Reasoning A form of reasoning from individual cases to general ones or from observed instances to unobserved ones.

Inequalities Statements indicating that two quantities are not equal, utilizing symbols > (greater than) or < (less than) and $\neq$.

Integers A set of numbers consisting of the whole numbers and their opposites {. . . –2, -1, 0, 1, 2 . . .}.

Inverse A related but opposite process or number such as multiplication being the inverse of division and 2/1 being the inverse of 1/2.

Irrational Numbers A set of numbers that cannot be represented as an exact ratio of two integers. For example, the square root of 2.

Iterative Processes In discrete math, a method of calculating an amount by using an initial value and applying a function repeatedly.

Linear Function A function that has a constant rate of change and can be modeled by a straight line.

Logarithm An alternative notation for expressing an exponent.

Logic A system of reasoning used to validate arguments.

Magnitude Size or quantity.

Manipulatives A wide variety of physical materials and supplies that students use to foster the learning of abstract ideas in mathematics.

Matrices A rectangular array of numbers or letters arranged in rows and columns.

Mean In statistics, the average obtained by dividing the sum of two or more quantities by the number of these quantities.

Measures of Central Tendency Numbers that communicate the "center" or "middle" of a set of data. The mean, median and mode are statistical measures of central tendency.

Median In statistics, the quantity designating the middle value in a set of numbers.

Mode In statistics, the value that occurs most frequently in a given series of numbers.

Model (noun) A display of concrete materials, objects or drawings.

Model (verb) Use of concrete materials, symbolic.

Monomial In algebra, an expression consisting of a single term such as $5y$.

Multiple A number into which another number may be divided with no remainder.

Nonstandard Measurement Measurement expressed in terms of objects such as paper clips, sticks of gun, shoes, etc.

Normal Curve In statistics, the distribution of data along a bell-shaped curve that reaches its maximum height at the mean.

Open Sentence A statement that contains at least one unknown. For example, $6 + x = 14$.

Parallelism The state of being parallel, not intersecting.

Parameter A quantity whose value varies with the circumstances of its application, such as the radius of a group of circles.

Permutations Ordered arrangements of a given number of items in a set.

Polynomial In algebra, an expression consisting of two or more terms such as $x^2 - 2xy + y^2$.

Primes Counting numbers that can only be evenly divided by two numbers which are the number itself and 1. For example, the numbers 2, 3, 5, 7.

Proportion An equality between ratios. For example, $2/6 = 3/9$.

Quadratic Function A function that has an equation of the form $y = Ax^2 + Bx + C$ where A does not equal 0.

Radian The size of the central angle of a circle when the arc length equals the radius.

Random Variable A quantity that can take any one of a number of unpredicted values.

Range In statistics, the difference between the greatest and smallest values in a set of data.

Ratio A comparison expressed as indicated division. For example, there is a ratio of three boys to two girls in our class (3/2, 3:2).

Rational Numbers Numbers that can be expressed as an exact ratio of two integers.

Real Numbers All rational and irrational numbers.

Rectangular Array An organized arrangement of square units (tiles).

Recurrence Relations In discrete mathematics, a value in a series is derived by applying a formula to the previous value.

Recursive Sequence In discrete mathematics, a series of numbers in which values are derived by applying a formula to the previous value.

Reflection In geometry, a transformation, also called a flip, that produces a mirror image of a geometric figure.

Rotation In geometry, a transformation that turns a figure about a point.

Sample A part of the total population. Used in statistics to make predictions about the characteristics of the entire group.

Scatter Plots A graph of the points representing a collection of data.

Scientific Notation A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10.

Similarity In geometry, objects or figures that are the same shape but not necessarily the same size.

Sine A trigonometric function that is defined as the ratio of the leg opposite the angle to the hypotenuse of its right triangle.

Skip Counting Counting by equal intervals.

Slides In geometry, a transformation where a figure moves in a given direction.

Square Root Two equal factors of a number. For example, 4 is the square root of 16.

Standard Deviation A statistic that measures the dispersion of a sample.

Stem-and-Leaf Plot A table utilizing digit(s) of a number as stems and the other digit(s) as leaves. For example, 5 | 7, 8 shows 57 and 58.

Survey Interview, questionnaire and/or polling.

Symmetry A correspondence in size, form and arrangement of parts on opposite sides of a plane, line or point. For example, a figure that has line symmetry has two halves that coincide if folded along its line of symmetry.

Synthetic Representation The geometric form as opposed to the algebraic representation of a figure.

Tangent A trigonometric function of an angle which is defined as the ratio of the lengths of the leg opposite to the leg adjacent to an angle in its right triangle. Also a line having one point in common with a curve.

Tessellations A mosaic formed by repetitions of a single shape.

Theoretical (mathematical) Relating to the probability of a given event, using mathematical relationships (e.g., the chance of a red side coming up on the flip of a two-colored counter is one in two or 1/2).

Transformation A geometric process for changing one figure into another.

Trigonometric Ratios The ratios of the lengths of pairs of sides in a right triangle, i.e., sine, cosine and tangent.

Trigonometry The branch of mathematics involving triangles that combines arithmetic, algebra and geometry. Trigonometry is used in surveying, navigation and physics.

Validity An argument that is correctly inferred or deduced from a premise.

Variability Numbers that describe how spread out a set of data is (e.g., range and quartile).

Variable A place holder in algebraic expressions. In $3x + y = 23$, x and y are variables.

Vector Quantity that has magnitude (length) and direction. It may be represented as a directed line segment (→).

Whole Numbers The counting numbers and zero {0, 1, 2, 3 . . .}.